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A Plug-and-Play Quaternion Message-Passing Module for Molecular Conformation Representation

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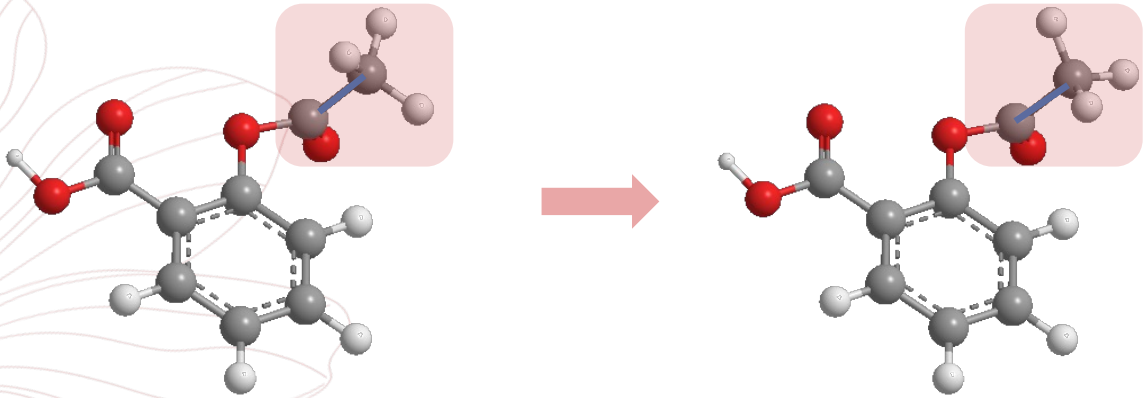
Background

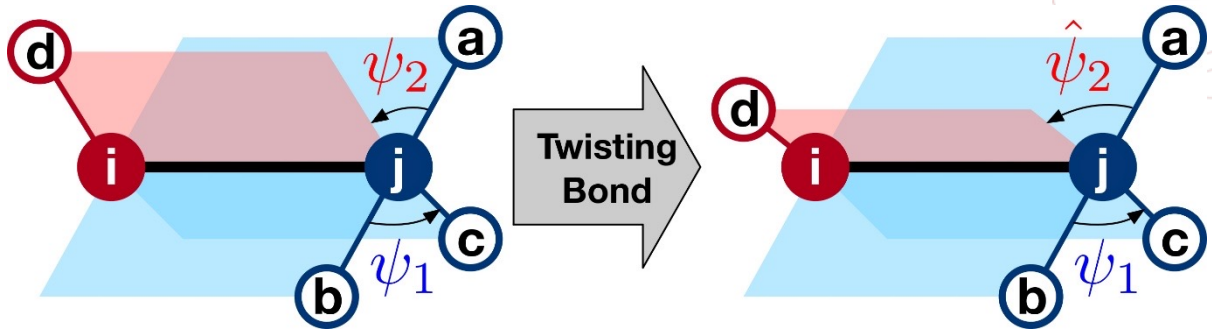
Molecular modeling

- **3D graph neural networks (GNNs)**
  - $G = (\mathcal{V}, \mathcal{E}, H^{(0)}, Z^{(0)}, X)$
  - $\mathcal{V}$: atoms, $\mathcal{E}$: edges, $H^{(0)}$: atom features, $Z^{(0)}$: bond features, X: coordinates.
  - Equivariant GNNs: coordinates, orientations, relative position…
  - Invariant GNNs: bond lengths, angles, dihedrals…
- **Conformation**



Background

Challenges for conformation representation

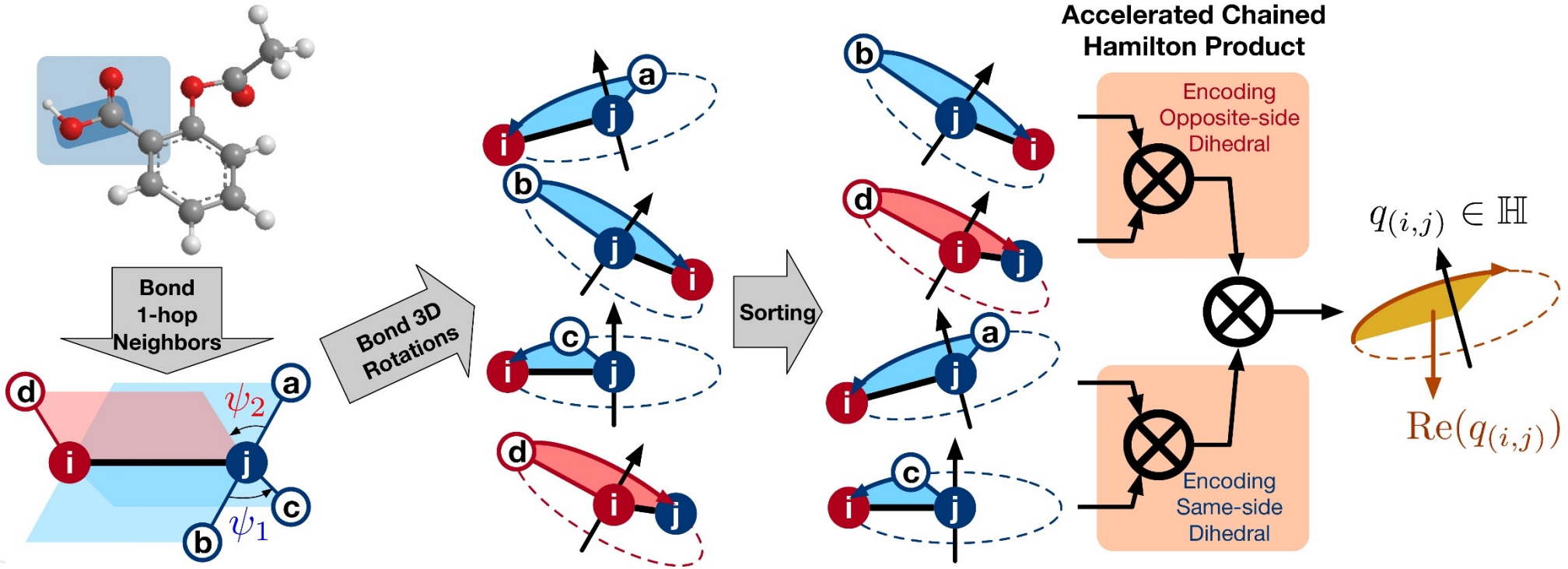


- **High computational complexity for complete representation.**
  - $\boldsymbol{\psi_1}$: same−side dihedral, $\boldsymbol{\psi_2}$: opposite−side dihedral.
  - A molecule with N atoms and D degrees on average: bond lengths $\mathrm{O}(ND)$, angles $\mathrm{O}(ND^2)$, , dihedrals $\mathrm{O}(ND^2)$ for $\boldsymbol{\psi_1}$ and $\mathrm{O}\left(\boldsymbol{ND^3}\right)$ for $\boldsymbol{\psi_2}$ **!!!**

Contributions

- We propose a **plug-and-play** quaternion message-passing (**QMP**) module to improve invariant GNNs in molecular conformation representation and analysis tasks
  - **QMP** achieves a **quaternion-based** mechanism to encode ψ_1 and ψ_2.
  - **QMP** is a **plug-and-play** design with small computational cost and one-line code.
  - Computational complexity of **QMP** is $\boldsymbol{O(NDlogD)}$.

Our Approach: Overview



- Quaternion Message Passing Module (**QMP**)
  - Encode the 3D rotations as a sequence of quaternions.
  - Aggregate the rotations by the chained Hamilton product of the quaternions.
  - Plug the real part into existing invariant GNNs.

Our Approach

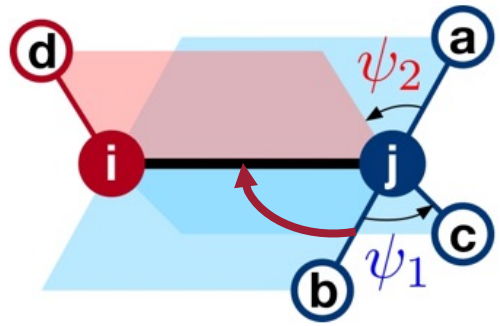
Encode rotations with quaternions

- **Quaternions:**
  - $q = s + \boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z \in \mathbb{H}$, s is the real part, and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the imaginary parts.
  - Represent rotations in 3D space. A rotation around the axis $\boldsymbol{u}$ with an angle θ.

$$q = [s, \boldsymbol{v}^T]^T = [\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\boldsymbol{u}^T]^T, ||\boldsymbol{u}||_2 = 1$$

- **Rotations:**
  - Given $(j,i), (j,k) \in \mathcal{E}$

$$u_{kji} = \frac{p_{jk} \times p_{ji}}{||p_{jk} \times p_{ji}||}, \theta_{kji} = arccos\left(\frac{\langle p_{jk}, p_{ji} \rangle}{||p_{jk}||_2 ||p_{ji}||_2}\right)$$



Our Approach

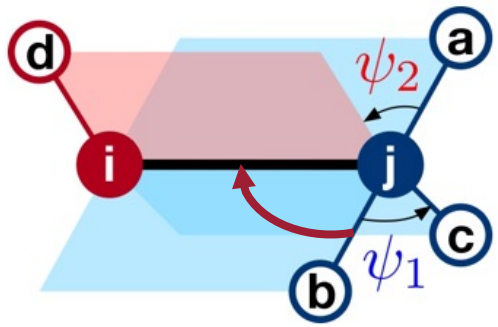
Angle-based Quaternion Sorting and Merging

- **Hamilton Product:**
  - For $q_1 = [s_1, \boldsymbol{v}_1{}^T]^T$ and $q_2 = [s_2, \boldsymbol{v}_2{}^T]^T$, the multiplication is:

$$q_1 \otimes q_2 = [s_1 s_2 - \langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle, \boldsymbol{v}_1 \times \boldsymbol{v}_2 + s_1 \boldsymbol{v}_2 + s_2 \boldsymbol{v}_1]$$

  - Hamilton product is **not commutative**.
- **Sorting and Merging**
  - For quaternion set of (j,i), $(i,j) \in \mathcal{E}$, select each top-K rotation angles and sort.
  - A quaternion sequence is $\mathcal{Q}_{(i,j)} = \left\{ q_k^{(i,j)} \right\} \in \mathbb{H}^{2K}$.

$$q_{(i,j)} = q_1^{(i,j)} \otimes q_2^{(i,j)} \otimes \cdots \otimes q_{2K}^{(i,j)} = \otimes_{q \in \mathcal{Q}_{(i,j)}} q$$



Our Approach

Plugging QMP into Invariant GNNs

- **Given a graph $G(\mathcal{V}, \mathcal{E}, H^{(0)}, Z^{(0)}, X)$, in the l-th message passing layer:**

$$z_{ij}^{(l+1)} = f_e(e_{ij}),$$

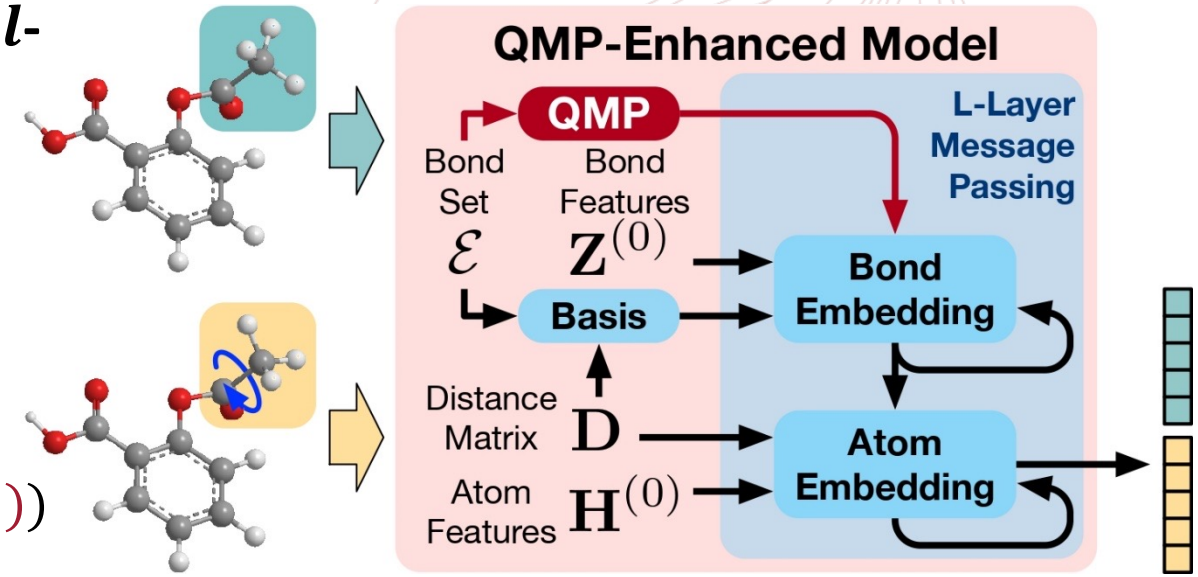
$$h_i^{(l+1)} = f_v(h_i^{(l)}, \sum_{j \in \mathcal{N}_i} z_{ij}^{(l+1)})$$

- **For DimeNet, DimeNet++, SphereNet**

$$e_{ij} = z_{ij}^{(l)} \oplus \sum_{k \in \mathcal{N}_{i \setminus j}} f_a(z_{ik}^{(l)}, b_D, \Theta, \Phi_s || Re(q_{(i,j)}))$$

- **For SchNet:**

$$e_{ij} = f_s(h_j^{(l)}) \odot f_{cf}(b_D) || Re(q_{(i,j)})$$



Theoretical Properties

Global SE(3)-Invariance

➢ **For $q_1 = [s_1, {\boldsymbol{v}_1}^T]^T$ and $q_2 = [s_2, {\boldsymbol{v}_2}^T]^T$, applying a rotation R on the rotation axes:**

$$[s_1, R(\boldsymbol{v}_1)^T]^T \otimes [s_2, R(\boldsymbol{v}_2)^T]^T$$

$$= [s_1 s_2 - \langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle, R(\boldsymbol{v}_1 \times \boldsymbol{v}_2 + s_1 \boldsymbol{v}_2 + s_2 \boldsymbol{v}_1)]$$

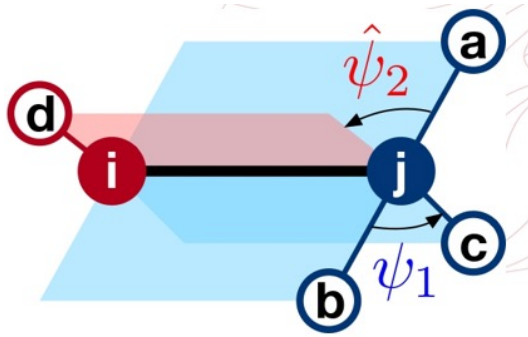
- Real part is global SE(3)-Invariant, imaginary part is SE(3)-Equivariant.

Sensitivity to local twisting

➢ **If only applies a rotation R on q_1. The real part of output is:**

$$s_1 s_2 - \langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle \neq s_1 s_2 - \langle R(\boldsymbol{v}_1), \boldsymbol{v}_2 \rangle$$

- Real part is sensitive to local twisting.



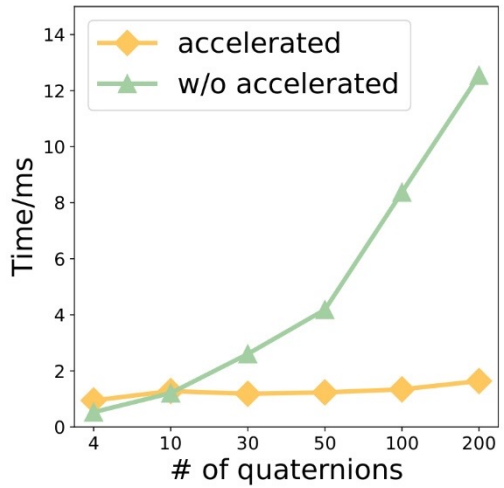
Theoretical Properties

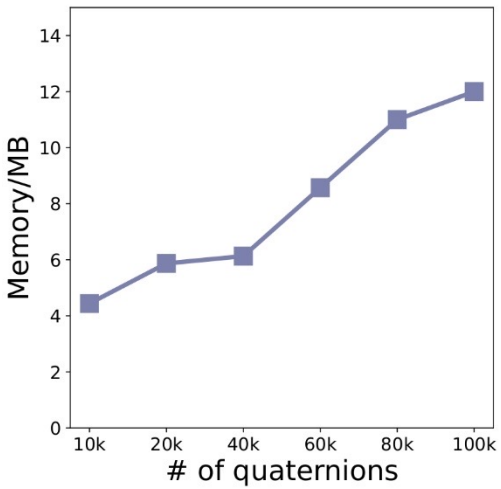
Mixed Encoding of Rotations and Dihedrals (ψ_1, ψ_2)

- **For real part**: $Re(q_1 \otimes q_2) = s_1 s_2 - \langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle$
  - $\langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle$ will compute $\langle \boldsymbol{u}_1, \boldsymbol{u}_2 \rangle$ which records the dihedral angle defined by the two axes.
  - If q_1 and q_2 from the same set: ψ_1. If q_1 and q_2 from the opposite set:ψ_2.

Computational Efficiency

- **QMP encodes dihedrals implicitly by merging a sequence of quaternions which only consider 1-hop neighbors $O(ND^2)$.**
- **Multi-thread reduction strategy to reduce to $O(NDlogD)$.**





Experiments

- **MD17 and MD17@CCSD.**

Table 1: Results in terms of force MAE ($\frac{\text{kcal}}{\text{mol}\,\mathring{A}}$). The best results are in bold. The second best results are with underlines. ↑ indicates the performance is improved. The results of sGDML and DimeNet are quoted from (Liu et al. 2022).

Dataset	Molecule	sGDML	DimeNet	SchNet	DimeNet++	SphereNet	Q-SchNet	Q-DimeNet++	Q-SphereNet
MD17	Aspirin	0.68	0.499	1.339	0.325	0.400	1.289↑	**0.316**↑	0.356↑
	Benzene	0.20	0.187	0.346	0.168	0.193	0.316↑	**0.151**↑	0.177↑
	Ethanol	0.33	0.230	0.738	0.150	0.181	0.468↑	**0.148**↑	0.169↑
	Malonaldehyde	0.41	0.383	1.559	0.263	0.379	1.540↑	**0.241**↑	0.321↑
	Naphthalene	0.11	0.215	0.723	**0.100**	0.159	0.521↑	0.106	0.138↑
	Salicylic acid	0.28	0.374	1.001	**0.231**	0.261	0.971↑	0.238	0.321
	Toluene	0.14	0.216	0.747	0.117	0.142	0.550↑	**0.107**↑	0.136↑
	Uracil	0.24	0.301	1.351	0.189	0.228	1.010↑	**0.178**↑	0.241
MD17@CCSD	Aspirin	-	-	1.471	0.387	0.458	1.298↑	**0.365**↑	0.475
	Benzene	-	-	0.352	0.049	0.061	0.287↑	**0.048**↑	0.063
	Ethanol	-	-	1.321	0.129	0.158	0.786↑	**0.108**↑	0.148↑
	Malonaldehyde	-	-	1.531	**0.211**	0.289	1.086↑	0.231	0.264↑
	Toluene	-	-	0.855	0.133	0.160	0.637↑	0.128↑	**0.052**↑

Experiments

- **OC20.**

Table 2: Results on OC20 IS2RE task in terms of energy MAE (eV) and the percentage of EwT of the ground truth energy. Performance is reported for models trained on the 10k training dataset.

Metrics		CGCNN	SchNet	DimeNet	DimeNet++	Q-SchNet	Q-DimeNet	Q-DimeNet++
Energy MAE	ID	0.9773	1.0480	0.9314	0.9261	1.0363↑	**0.9132**↑	0.9177↑
	OOD Ads	0.9818	1.0450	1.0720	0.9400	1.0640	0.9865↑	**0.9295** ↑
	OOD Cat	0.9269	1.0630	0.8945	0.8828	1.0370 ↑	**0.8753**↑	0.8816↑
	OOD Both	0.8828	1.0076	0.9643	0.8572	0.9646 ↑	0.9317↑	**0.8513**↑
	Average	0.9422	1.0409	0.9656	0.9015	1.0255 ↑	0.9267↑	**0.8950**↑
EwT (%)	ID	1.84	1.58	1.80	1.79	1.59↑	1.84↑	**1.91**↑
	OOD Ads	1.72	1.53	1.62	1.67	1.52	**1.85**↑	1.79↑
	OOD Cat	1.93	1.56	1.93	1.81	1.46	1.96↑	**2.13**↑
	OOD Both	1.69	1.48	1.57	**1.80**	1.61↑	1.70↑	1.75↑
	Average	1.80	1.54	1.73	1.77	1.55↑	1.84↑	**1.90**↑

Experiments

- **Ablation study.**

Table 3: Effects of ablating components of QMP.

QMP Components			Force MAE	
j-side	i-side	sorting	Aspirin	Benzene
✓	X	✓	0.323	0.152
X	✓	✓	0.360	0.166
✓	✓	X	0.390	0.173
✓	✓	✓	**0.316**	**0.151**

- **Potential on equivariant GNNs.**

Table 4: Mean Squared Error ($\times 10^{-2}$) for the future position estimation in the N-body system experiment.

Method	SE(3) Trans.	TFN	EGNN	Q-EGNN
MSE	2.440	1.550	0.724	**0.668**

Conclusion

- An efficient and effective quaternion message-passing module (QMP) for molecular conformation representation and analysis.
- QMP achieves global SE(3)-invariance and enhances the sensitivity to local bond twisting simultaneously.
- With a small computational cost, we can plug this module into most existing invariant GNNs by one-line code.
- In the future, we would like to utilize the imaginary part to enhance equivariant GNNs. Additionally, we plan to design more hypercomplex neural networks based on Clifford Algebra

Code: https://github.com/AngxiaoYue/QMP

Thank You for listening!